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Remarks on generalized rings of quotients, III

By

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Introduction. Let R be a commutative ring with a unit element. In [10], Lazard has shown that there is a unique (up to isomorphism over R) maximal ring extension $M(R)$ of R such that $M(R)$ is R-flat and the canonical injection j of R into $M(R)$ is an epimorphism in the category of commutative rings with units, that is, if f and g are ring-homomorphisms of $M(R)$ into a commutative ring R' with a unit element such that $fj=gj$, then $f=g$ (we always assume that a unit element is mapped to a unit element). $M(R)$ is also characterized by the property that if S is an overring of R such that S is R-flat and the canonical injection of R into S is an epimorphism, then S is isomorphic to a subring of $M(R)$ which is R-flat.

On the other hand, a maximal quotient ring $Q(R)$ is defined for an arbitrary (not necessarily commutative) ring R as a maximal rational extension (the definition of a rational extension is stated in §1) of R in an injective envelope*) of R as an R-module and is unique up to isomorphism over R. In the case where R is commutative, $Q(R)$ is also commutative and contains the total quotient ring $T(R)$ of R (see [5] or [8]).

Bourbaki has given a general method to construct "rings of

*) An injective envelope of an R-module M is an injective R-module which is an essential extension of M (see Def. 2 in §1), and is unique up to isomorphism. If an R-module N is an essential extension of M, then there is an injective envelope of M which contains N.