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Rational equivalence of 0-cycles on surfaces

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We will consider in this note 0-cycles on a complete non-singular algebraic surface F over the field $\boldsymbol{C}$ of complex numbers. We will use the language of schemes, and every scheme will be assumed separated and of finite type over $\boldsymbol{C}$. In a very extensive set of papers, Severi set up and investigated the concept of rational equivalence (cf. [2], [3], [4], [5] among many others). It is not however very easy to find a precise definition in Severi's work, and there was a good deal of discussion on this point at the International Congress of 1954. A much more elementary approach was worked out by Chevalley in his seminar "Anneaux de Chow" [1]. For 0-cycles on F, the most elementary definition is this:

Let Σ_n = group of permutations on n letters.

Let $S^n F = F^n/\Sigma_n$, the n^{th} symmetric power of F

$$\underset{\text{as set}}{\cong} \{A \mid A \text{ effective 0-cycle of degree } n \text{ on } F\}.$$

Definition: 2 0-cycles A_1, A_2 of degrees n_1 and n_2 are *rationally equivalent* if $n_1 = n_2$ and $\exists$ a 0-cycle B of degree m such that $A_1 + B$ and $A_2 + B$ are effective, corresponding to points $x_1, x_2 \in S^{n+m}F$, and $\exists$ a morphism $f: \boldsymbol{P}^1 \to S^{n+m}F$ such that $f(0) = x_1$, $f(\infty) = x_2$.

Definiton: $A_0(F)$ = group of all zero-cycles of degree 0 on F modulo rational equivalence.