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Simplical cohomology and n-term extensions of algebras

By

Akira Iwai

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Simplicial method is very useful in discussing (co)homology theory in a non-abelian category. M. André [1] and J. Beck [2] investigated the simplicial cohomology $H^*(A, M)$ of a commutative algebra A over a basic ring K with coefficients in an A-module M.

The purpose of the present paper is to interpret the cohomology $H^*(A, M)$. Our interpretation of $H^*(A, M)$ is an analogy of that of the functor Ext* by N. Yoneda [9].

It has been known that the 0-th cohomology group $H^0(A, M)$ is isomorphic to the module $\mathrm{Der}_K(A, M)$ of K-derivations of A into M, and the first cohomology group $H^1(A, M)$ is in $1-1$ correspondence with the set $\mathrm{Ex}^1(A, M)$ of isomorphic classes of 1-term extensions of A by M. N. Shimada and others [8] have shown that the second cohomology group is in $1-1$ correspondence with the set $\mathrm{Ex}^2(A, M)$ of equivalence classes of 2-term extensions of A by M in the sense of S. Lichtenberg and S. Schlessinger [7] (or in M. Gerstenhaber [5]).

We start with the definitions of quasi-simplicial algebras and the simplicial cohomology. Let $\mathcal{A}$ be the category of associative commutative algebras with unit over a basic ring K. Denote by $(\mathcal{A}, A)$ the category of morphisms in $\mathcal{A}$ with range A.