

On tensor products Gorenstein rings

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Introduction

Among local rings there is a hierarchy: regular \Rightarrow complete intersection \Rightarrow Gorenstein \Rightarrow Macaulay. These concepts are extended to non-local rings; for example a ring A is regular if for all prime ideal \mathfrak{p} of A , $A_{\mathfrak{p}}$ is a regular local ring.

In this paper, we shall investigate if these properties are conserved under tensor product operations. It is well known that the tensor product $B \otimes_A C$ of regular rings are not regular in general, even if we assume A , B and C are fields.

But it can be shown that under a suitable condition tensor products of regular rings are complete intersections. For Macaulay rings and Gorenstein rings, it is proved implicitly in [2], that tensor product $B \otimes_A C$ of Macaulay rings are again Macaulay if we assume B is A -flat and C is finitely generated over A , and we shall show that the same is true for Gorenstein rings.

Part I, which is the main part of the present paper, was written by the first author. If one assume that A is a field, then the treatment is much simpler and we have a similar result under a weaker finiteness condition. The case was given by the second and the third authors and is the contents of Part II. On the other hand, the case of complete tensor products over a field was observed by the last author, who wrote Part III.