On mixed problems for hyperbolic systems of first order with constant coefficients

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§1. Introduction

The mixed problem for linear hyperbolic equations with constant coefficients in a quarter space has been treated by S. Agmon [1], R. Hersh [2] and L. Sarason [7]. S. Agmon treated single higher order equations and R. Hersh and L. Sarason the first order systems.

In the present paper we consider the mixed problem for hyperbolic systems of first order with principal part having constant coefficients:

(1.1)
$$\begin{cases} \frac{\partial}{\partial t}u + A\frac{\partial}{\partial x}u + \sum_{j=1}^{n}B_{j}\frac{\partial}{\partial y_{j}}u = f(t; x, y) \\ u(0; x, y) = 0 \\ Pu(t; 0, y) = 0 \end{cases}$$

in a quarter space $\{(t, x, y); t > 0, x > 0, y \in \mathbb{R}^n\}$, where *u* is a *N*-vector, *A*, *B_i* $(j=1, 2, \dots, n)$ are *N*×*N*-constant matrices and *A* is non-singular, and *P* is $m \times N$ -constant matrix and its rank *m*. Already L. Sarason [7] also gave a priori estimates for the system (1, 1), our approach is slightly different from Sarason's one. Moreover we treat the problem (1, 1) under less stringent conditions in some sense.

Our argument is based on Wiener-Hopf's method. Taking Laplace transformation in t and Fourier transformation in y, the ploblem (1.1) becomes to the problem of a system of ordinary differential equations