

On differential systems, graded Lie algebras and pseudo-groups

By

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Dedicated to Prof. Atsuo KOMATSU
for his 60th birthday

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Introduction

A differential system (or Pfaffian system) D on a manifold M is a law which assigns to every point $x \in M$ a subspace $D(x)$ of a given dimension of the tangent space $T_x(M)$ to M at x and which is differentiable in a suitable sense, or it may be simply defined as a subbundle of the tangent bundle $T(M)$ of M . Let D (resp. D') be a differential system on a manifold M (resp. M'). Then a diffeomorphism φ of M onto M' is called an isomorphism of (M, D) onto (M', D') if it induces a bundle isomorphism of D onto D' , i.e., $\varphi_*D(x) = D'(\varphi(x))$ at each $x \in M$.

The geometry of differential systems may be described as usual as a geometry of linear group structures (G -structures) whose structure group G is of infinite type and even not elliptic. This fact makes the geometry rather difficult to be studied on the basis of the usual theory of linear group structures. (By the usual theory, we here mean the local theory as appears in Singer and Sternberg [9] and the global theory (cf. Ruh [8] and Ochiai [7]) based on the theory of elliptic differential equations.) For instance, it seems to us that a direct use of the usual theory fails to give any finiteness theorem on the automorphism groups of differential systems. The same remark holds for other