J. M. Kyoto Univ.
10-3 (1970) 475-492

On a small drift of Cauchy process

By

Masaaki TSUCHIYA

(Communicated by April 14, 1970)

Introduction

The purpose of this paper is to show the existence and the uniqueness of a Markov process corresponding to the operator $Af(x)=a(x)f'(x)+Bf(x)$, where B is the infinitesinal generator of 1-dimensional symmetric Cauchy process and a is a bounded measurable function. If a is Lipschitz continuous, this problem can be solved as a particular case of the general theory of stochastic integral equations due to K. Ito. The difficulty arises if a is not Lipschitz continuous. In this paper, by making use of the method initiated by D. W. Stroock and S. R. S. Varadhan [13], we shall solve the above problem when a is a measurable function which lies in a sufficiently small neighborhood (with respect to the supremum norm) of a constant function. The problem has important meaning in the so-called boundary problems of diffusion processes since the process corresponding to A becomes the Markov process on the boundary of a Brownian motion with an oblique reflection on the upper half plane. As for the Markov process on the boundary of diffusion processes and its role in the boundary problems of diffusion processes, we refer to M. Motoo [8], K. Sato-T. Ueno [11] and N. Ikeda [2].

Now we summarize the content of this paper. In §1, we prepare the notations and some preliminary facts. In §2, we construct the operator K_λ such that $K_\lambda(\lambda I-A)=I$. Formally, K_λ is expressed as

$$K_\lambda = G_\lambda(I-T_\lambda)^{-1},$$