On stochastic differential equations for multi-dimensional diffusion processes with boundary conditions

By

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Introduction

In this paper, we shall discuss on stochastic differential equations for sample functions of multi-dimensional diffusion processes with boundary conditions. On this subject, important works were given by Ikeda [2] and Skorohod [7]. Ikeda discussed the construction of two dimensional diffusion processes with Wentzell's boundary conditions on a unit disk using the known property of one dimensional reflecting Bessel processes. Skorohod discussed the stochastic differential equations for reflecting diffusion processes. Our main objective of the present paper is to unify these two works. We shall formulate the stochastic differential equations with boundary condition in Definition 1 and show the existence and the uniqueness of solutions in Theorem 1, which is our main result. The uniqueness obtained there is that in the sense of the probability law. It seems difficult to give a natural formulation of the pathwise uniqueness except some special cases. As a consequence, we can construct, in a purely probabilistic way, a class of diffusion processes with Wentzell's boundary conditions. In analytic way, such a problem has been discussed by Sato-Ueno [6] and Bony-Courrège-Priouret [1].