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A theorem of Gutwirth

By

Masayoshi NAGATA

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The following fact was proved by Gutwirth[1] in the classical case: Let D be a line on $\boldsymbol{P}^2$ and consider the affine plane $S=\boldsymbol{P}^2-D$. Assume that C is an irreducible curve defined over a ground field K and of degree, say d, on $\boldsymbol{P}^2$ such that $C\cap S$ is biregular to an affine line. Then $C\cap D$ contains a unique ordinary point, say P. If we look at also infinitely near points, then all of singular points, say $P_1, \cdots, P_n$ are arranged so that (i) $P=P_1$ and (ii) each P_{i+1} is an infinitely near point of P_i of order 1. Let m_i be the effective multiplicity of P_i on C (that is, the multiplicity of P_i on the proper transform of C by successive quadratic dilatations with centers $P_1, \cdots, P_{i-1}$). On the other hand, let $f(x, y)$ be the irreducible polynomial which defines $C\cap S$ in the affine coordinate ring $K[x, y]$ of S. Then

Theorem. *Consider the linear system L of curves of degree d on $\boldsymbol{P}^2$ which goes through $\sum m_i P_i$. If $\dim L\geq 1$, then d is a multiple of $d-m_1$.*

This fact implies also, under the same assumption, that there is a polynomial $g(x, y)$ such that $K[x, y]=K[f, g]$.

The purpose of the present paper is to give a proof of the above theorem without any restriction on the ground field K. We add also

1) A. Gutwirth, An inequality for certain pencils of plane curves, Proc. Amer. Math. Soc. Vol. **12** (1961) pp. 631-639