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The decomposition of L^2 $(\Gamma\backslash SL(2, R))$ and Teichmüller spaces

By

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§0. Introduction

Let H be the complex upper half plane and let Γ be a discrete subgroup of the group G of conformal automorphisms of H. We assume that $\Gamma\backslash H$ is compact.

For each unitary matrix representation χ of Γ, we consider an eigenvalue problem (called the (Γ, χ)-eigenvalue problem in §1,) following [S]. The spectra of this eigenvalue problem and its generalizations have been investigated since the famous paper of A. Selberg [S] appeared in 1956. But, at present, not much is known even in the above special case.

In this paper, we want to study "How do the spectra of (Γ, χ)-problem behave when Γ varies?"

There is another (more group-theoretical) interpretation of our problem. We give it in the following.

Let G, Γ, χ be as above and let $U = \operatorname{Ind}_{\Gamma\uparrow G} \chi$ be the unitary representation of G induced from χ. As is well known, U can be decomposed into the discrete sum $\sum_i \oplus U_i$ of irreducible unitary representations U_i of G. We call the set $S_U = \{U_i; i = 1, 2, \cdots\}$ the spectra of $U = \operatorname{Ind}_{\Gamma\uparrow G} \chi$ and decompose it into the disjoint union of two subsets, the C-part S_U^C and the D-part S_U^D, where S_U^C consists of those elements of S_U con-