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Commutative rings which are locally Noetherian

By

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It is well known that if R is a Noetherian commutative ring, then R_M, the localization of R at each maximal ideal M, is also Noetherian. Any almost Dedekind domain[1)] which is not Dedekind provides an example for which the converse fails [1, App. 1, Ex. 1]. We shall say that a ring R is *locally Noetherian* provided R_M is Noetherian for each maximal ideal M of R. The goal of this paper is to characterize those locally Noetherian rings which are also Noetherian.

Throughout this paper R will denote a commutative ring with identity. Our notation and terminology are essentially that of [1] with the following exception: if S is a multiplicatively closed subset of R and if A is an ideal of R, then we shall denote by AR_S the extension of A to R_S.

1. The Characterization Theorem

Let A be any ideal of R and let S be the set of elements of R which are not zero divisors modulo A. Then S is multiplicatively closed and $A \cap S = \emptyset$. Any prime ideal containing A which is maximal with respect to missing S is called a *maximal prime divisor* of A. A prime ideal P of R is called a *prime divisor* of A if there is a multi-

1) If D is an integral domain with identity, then D is said to be an *almost Dedekind domain* if D_M is a Noetherian valuation ring for each maximal ideal M of D [1, p. 408].