

On Neggers' numbers of discrete valuation rings

By

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The purpose of this note is to show the converse of Theorem 3 in [3], that is,

Theorem. *Let R be a complete discrete valuation ring of unequal characteristic with a prime element u and with a coefficient ring P . Let K and K^* be quotient fields of R and P , respectively. If the Neggers' number $\Delta_{K|K^*}(u) < 1$, there exists a coefficient ring \bar{P} of R such that $\Omega_{R|\bar{P}}$ is not isomorphic to $\Omega_{R|P}$.*

In this paper we use the same notations and terminology as in [3]. Then, together with results in [1] and [3], we obtain various characterizations of the property that $\Delta_{K|K^*}(u) \geq 1$:

Corollary. *The following conditions are equivalent.*

- (1) $\Delta_{K|K^*}(u) \geq 1$ for a choice of P and u .
- (2) $\Delta_{K|K^*}(u) \geq 1$ for every choice of P and u .
- (3) Every derivation in $\text{Der}(R, R)$ induces a derivation in $\text{Der}(R/m, R/m)$.
- (4) Every derivation in $\text{Der}(R/m, R/m)$ is induced by a derivation in $\text{Der}(R, R)$.
- (5) $\Omega_{R|P}$ is determined independently of the choice of P , up to