J. Math. Kyoto Univ. (JMKYAZ)
11-2 (1971) 357-372

On the initial-value problems with data on a double characteristic

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(Received December 10, 1970)

§1. Introduction

Let us consider a linear partial differential equation

$$a(x, \partial)\, u(x) \equiv \sum_{|\nu| \leqq m} a_\nu(x) \partial^\nu u(x) = f(x), \quad x \in R^n. \text{ *)} \tag{1.1}$$

Let S be a hypersurface in R^n defined in a neighborhood of $x_0 \in S$ by

$$\varphi(x) = 0;\ \varphi_x(x) \equiv (\partial_1\varphi(x), \partial_2\varphi(x), \cdots, \partial_n\varphi(x)) \neq 0. \tag{1.2}$$

We say that S is a double characteristic hypersurface of the operator $a(x, \partial)$, if φ satisfies the following conditions:

$$\begin{cases} h(x, \varphi_x) = 0, & x \in S, \\ \dfrac{\partial}{\partial \xi_i} h(x, \varphi_x) = 0, & x \in S,\ i = 1, 2, \cdots, n, \\ \sum_{i,j} \left| \dfrac{\partial^2}{\partial \xi_i d\xi_j} h(x, \varphi_x) \right| \neq 0, & x \in S, \end{cases} \tag{1.3}$$

where $h(x, \xi) = \sum_{|\nu| = m} a_\nu(x) \xi^\nu$.

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*) In this article we use the following notations:
∂ stands for $\dfrac{\partial}{\partial x}$ and ∂_i, ∂_x, ∂_y stand for $\dfrac{\partial}{\partial x_i}$, $\dfrac{\partial}{\partial x}$ and $\dfrac{\partial}{\partial y}$ respectively. In the case where $\partial_y^\alpha u(y)$, we often represent it simply by $\partial^\alpha u(y)$.