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Notes on classification of Riemann surfaces

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1. Introduction.

To classify Riemann surfaces (simply, "surfaces") by Hardy classes seems to have long been an open question. Recently Heins solved this problem thoroughly in his Springer lecture note [2, pp. 34–51]. The objective of the present article is to show that surfaces R of class O_{H_p} $(0<p<\infty)$ or of class O_{AB} or of class O_{LA} [2, p. 35] are characterized by a certain topological property of analytic functions on R, where O_{H_p} denotes the totality of surfaces R on which Hardy class $H_p(R)$ contains only constant members. The reader should know what is meant by O_{AB}, O_{AD} and O_{HD} [1, pp. 200 and 198].

A complex-valued harmonic function f on a surface R is said to be open if $w=f(P)$, $P\in R$, carries open subsets of R to those of the w-plane. Given a surface R, we denote by $\mathscr{L}(R)$, $\mathscr{H}_p(R)\,(0<p<\infty)$, $\mathscr{B}(R)$ and $\mathscr{D}(R)$ the classes of open harmonic functions f on R such that $\log^+|f|$ has a harmonic majorant on R, $|f|^p$ has a harmonic majorant on R, f is bounded on R and f has finite Dirichlet integral on R, respectively. We denote by $O_X(X=\mathscr{L}, \mathscr{H}_p, \mathscr{B}, \mathscr{D})$ the class of surfaces R on which $X(R)$ is *empty*. Then we have

(1) $$O_{H_p}=O_{\mathscr{H}_p} \quad \text{for } 0<p<\infty.$$

(2) $$O_{AB}=O_{\mathscr{B}} \quad \text{and} \quad O_{LA}=O_{\mathscr{L}}.$$