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Algebra of stable homotopy of Z_p-spaces and applications

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Introduction

The stable homotopy classes of maps: $\sum^{t+n}X \rightarrow \sum^{n}Y$ will be denoted by $\pi_t^S(X; Y)$. When $X=Y$, $\mathscr{A}_*(X)=\sum \mathscr{A}_t(X)$, $\mathscr{A}_t(X)=\pi_t^S(X; X)$ forms a graded ring with the composition as the multiplication. p denotes an odd prime and $M=S^n \cup e^{n+1}$ a Moore space of type (Z_p, n). We call a space X a Z_p-space if $\mathscr{A}_*(X)$ is an algebra over Z_p or equivalently $M \wedge X$ is the same homotopy type of $\sum^n X \vee \sum^{n+1} X$ (n: large). Then $\pi_t^S(M \wedge X; M \wedge Y)$ is decomposed into $\pi_{t+1}^S(X; Y) \oplus \pi_t^S(X; Y) \oplus \pi_t^S(X; Y) \oplus \pi_{t-1}^S(X; Y)$. For given $\gamma \in \pi_t^S(X; Y)$, the smash product $1_M \wedge \gamma$ is decomposed to $\theta(\gamma) \oplus \gamma \oplus \gamma \oplus 0$, and we have a linear map

$$\theta: \pi_t^S(X; Y) \rightarrow \pi_{t+1}^S(X; Y)$$

This θ is a derivation:

$$\theta(\gamma\gamma')=\theta(\gamma)\cdot\gamma'+(-1)^{\deg\gamma}\gamma\theta(\gamma')$$

and if the spaces satisfy a sort of associativity then θ is a differential $\theta\theta=0$ (Theorem 2.2).

On the other hand, for a given $\xi \in \mathscr{A}_t(M)$ the decomposition $\xi \wedge 1_X = \lambda_X(\xi) \oplus$(other terms) defines a linear map

$$\lambda_X: \mathscr{A}_t(M) \rightarrow \mathscr{A}_{t+1}(X).$$

The basic property of this operation is the following commutation