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On the Riemann-Roch theorem on open Riemann Surfaces

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Introduction

To generalize the classical theory of algebraic functions to open Riemann surfaces, much effort has been made in the last three decades. As for Riemann-Roch theorem and Abel's theorem, similar formulations as classical were obtained by L. Ahlfors [2] [3] [4], Y. Kusunoki [6], B. Rodin [15] and H.L. Royden [16] for some class of open surfaces. The results but for [6] are described in terms of *distinguished* harmonic differentials introduced by Ahlfors. Although restrictions for surfaces are not explicitly mentioned, they seem to be meaningful only for surfaces with small boundaries, say, those of class O_{KD}. Otherwise, a single-valued meromorphic function whose differential is distinguished would reduce to a constant. As was pointed out by R.D.M. Accola [1], the same situation occurs if the surface belongs to the class $O_{HD}-O_G$. For surfaces of class $O_{KD}-O_{HD}$, it seems yet unknown whether or not non-constant meromorphic function f exists such that df is distinguished. While, the results by Y. Kusunoki [7] [8] [9] are meaningful for general surfaces. His results are given in terms of *canonical semiexact* differentials and functions introduced by himself, which have some restrictions only in their real parts. M. Mori [13] pointed out that canonical semiexact differentials are identical with meromorphic differentials whose real parts are distinguished (in the real sense). Recently