Topological submanifolds and homology classes of a topological manifold

By

Masahisa ADACHI[*]

This note is devoted to the problem of the realization of homology classes of a topological manifold by topological submanifolds. Firstly the C^∞-case of this problem was studied by R. Thom [6], and secondly the PL-case in [1], [2].

The present study is founded on the Kirby-Siebenmann's transversality theorem [3]. We shall apply R. Thom's method [6] to topological manifolds.

The author is grateful to Professor Y. Saito and Mr. T. Matumoto for their kind criticisms.

1. Statement of the results

We shall obtain the following results.

i) *Homology classes* mod 2.

Theorem 1. *Let V^n be a closed topological manifold of dimension n, and $n \neq 4$. Then the following homology classes mod 2 are realizable by topological submanifolds which have normal vector bundles in V^n:*

(a) $H_{n-1}(V^n, \mathbf{Z}_2)$, *for $n \neq 5$, $n \geq 1$;*

(b) $H_{n-2}(V^n, \mathbf{Z}_2)$, *for $2 \leq n < 6$;*