

# Cluster sets at ideal boundary points

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## Introduction

The theory of cluster sets in plane regions has been studied in greater detail. To extend that theory to the case of Riemann surfaces it is natural to consider some kinds of compactifications of Riemann surfaces and define, in analogy of plane case, several kinds of cluster sets at ideal boundary points of the compactification. Then, one can expect that recent systematic studies of compactifications will give effective tools for the study of cluster sets at ideal boundary points.

In this paper we study cluster sets of an analytic mapping at Martin boundary points of a hyperbolic Riemann surface, where the cluster sets are defined with respect to Martin topology and to the fine topology. Especially in the case of a Fatou mapping, Wiener compactification, of which Martin compactification is a quotient space, is used to represent some of our cluster sets as sets of values of the mapping on certain subsets of its boundary (§3).

For compactifications and related notions we refer to Constantinescu-Cornea [4].

## §1. Definitions.

We consider a non-constant analytic mapping  $f$  of a Riemann surface  $R$  into  $R'$ .  $R$  is assumed hyperbolic in the sequel unless otherwise stated. Let  $R^*$  be Martin compactification of  $R$  and  $R'^*$  a metrizable and resolute compactification of  $R'$ . We denote by  $\mathcal{A}$