On the nilpotence of the hypergeometric equation

By

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Introduction

Let $T$ be an arbitrary scheme, $S$ a smooth $T$-scheme and $\mathcal{M}$ a quasi-coherent $\mathcal{O}_S$-module. A $T$-connection on $\mathcal{M}$ is by definition a homomorphism of $\mathcal{O}_S$-modules:

$$\nabla: \mathcal{D}er_{\mathcal{O}_T}(\mathcal{O}_S, \mathcal{O}_S) \rightarrow \mathcal{E}nd_{\mathcal{O}_T}(\mathcal{M})$$

which satisfies the "product formula":

$$\nabla(D)(sm) = s\nabla(D)(m) + D(s)m$$

for sections $D$ of $\mathcal{D}er_{\mathcal{O}_T}(\mathcal{O}_S, \mathcal{O}_S)$, $s$ of $\mathcal{O}_S$ and $m$ of $\mathcal{M}$ over an open subset $U \subseteq S$. A section $m$ of $\mathcal{M}$ over $U$ is called horizontal if $\nabla(D)(m) = 0$ for all $D$'s, derivations on open subsets of $U$. Both $\mathcal{D}er_{\mathcal{O}_T}(\mathcal{O}_S, \mathcal{O}_S)$ and $\mathcal{E}nd_{\mathcal{O}_T}(\mathcal{M})$ are $\mathcal{O}_T$-Lie-algebras via the commutator bracket. The connection is called integrable if it is a Lie-algebra homomorphism. The obstruction to a connection being integrable is the curvature homomorphism $K: \bigwedge^2 \mathcal{D}er_{\mathcal{O}_T}(\mathcal{O}_S, \mathcal{O}_S) \rightarrow \mathcal{E}nd_{\mathcal{O}_S}(\mathcal{M})$ defined by $K(D \wedge D') = [\nabla(D), \nabla(D')] - \nabla([D, D'])$. Henceforth we will deal only with integrable connections.

A horizontal morphism $\phi$ between modules with connection