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A role of Fourier transform in the theory of infinite dimensional unitary group

By

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§ 0. Introduction

In the investigation of the sample function properties of the complex Brownian motion, the Fourier analysis plays a dominant role to discuss generalized harmonic analysis or frequency problems. For our purpose, we prefer the complex white noise rather than the complex Brownian motion, since probabilistic properties in question of the former are somewhat simpler than those of the latter. Thus, we are naturally led to discuss the Fourier transform of sample functions of the complex white noise which are, of course, generalized functions.

Our discussion, therefore, starts with the general set-up of the *complex white noise* (§1.). Let E be a σ-Hilbert nuclear space which is included densely in $L^2(R^1)$, and let E_c be the complexification of E. The complex white noise gives a probability measure ν, which is Gaussian, on the space E_c^* the conjugate space of E_c. We then, in § 1, come to the *infinite dimensional unitary group* $U(E_c)$ which is the collection of all the linear transformations on E_c leaving the L^2-norm invariant. Here, it should be noted that the basic space E_c must be chosen so that the Fourier transform is a linear isomorphism of the space E_c; namely the Fourier transform is a member of $U(E_c)$. We also wish to topologize the space E_c by using the differential operator D which is invariant (up to multiplicative constant) under the