

On a canonical operation of $\mathfrak{sl}(2m)$ on the exterior algebra of the vector space of complex $m \times n$ -matrices

By

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1. We mean by Z_{ip} ($1 \leq i \leq m, 1 \leq p \leq n$) independent complex variables and denote by E the exterior algebra generated by $dZ_{ip}, d\bar{Z}_{jq}$ ($1 \leq i, j \leq m; 1 \leq p, q \leq n$). For brevity we put

$$\theta_{ip} = dZ_{ip}, \quad \bar{\theta}_{jq} = d\bar{Z}_{jq} \quad (1 \leq i, j \leq m; 1 \leq p, q \leq n).$$

Operators L_{ij}, A_{ij} acting on E are defined by

$$L_{ij} = \sqrt{-1} \sum_{p=1}^n e(\theta_{ip}) e(\bar{\theta}_{jp}),$$

$$A_{ij} = -\sqrt{-1} \sum_{p=1}^n i(\theta_{ip}) i(\bar{\theta}_{jp})$$

where $e(\xi)\eta = \xi \wedge \eta$ and $i(\xi)\eta$ is the inner product of ξ with η with respect to the metric

$$2 \sum_{i=1}^m \sum_{p=1}^n (dZ_{ip}, d\bar{Z}_{ip}).$$

In the present note we shall show that there exists a representation ρ of Lie algebra $\mathfrak{sl}(2m)$ on E such that

$$\rho \left(\begin{array}{c|c} 0 & (a_{ij}) \\ \hline 0 & 0 \end{array} \right) = \sum_{i,j=1}^m (-1)^{j+1} a_{ij} L_{ij},$$

$$\rho \left(\begin{array}{c|c} 0 & 0 \\ \hline (b_{ij}) & 0 \end{array} \right) = \sum_{i,j=1}^m (-1)^{j+1} b_{ji} A_{ij}.$$