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Simple groups of conjugate type rank 5

By

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1. Introduction

Let $\mathfrak{G}$ be a finite group, $I(\mathfrak{G})$ the set of indices of centralizers of non-central elements of $\mathfrak{G}$ in $\mathfrak{G}$, and r the number of elements in $I(\mathfrak{G})$. r is called the conjugate type rank of $\mathfrak{G}$. We introduce an ordering in $I(\mathfrak{G})$ as follows: let a and b be two elements of $I(\mathfrak{G})$. Then $a>b$ if and only if a divides b. Let k be the number of maximal elements in $I(\mathfrak{G})$. Then $\mathfrak{G}$ is called k-headed. We form a graph $C(\mathfrak{G})$ of $\mathfrak{G}$ as follows: the points of $C(\mathfrak{G})$ are the elements of $I(\mathfrak{G})$. The (oriented) edge ab of $C(\mathfrak{G})$ exists, where a and b are points of $C(\mathfrak{G})$, if and only if $a>b$. We denote the edge ab by $\begin{matrix} a \\ \uparrow \\ b \end{matrix}$. $C(\mathfrak{G})$ is called the conjugate type graph of $\mathfrak{G}$. The centralizer of any non-central element of $\mathfrak{G}$ in $\mathfrak{G}$ corresponding to an isolated point of $C(\mathfrak{G})$ is called free.

An obvious problem is as follows: Let r be a given positive integer. Then classify all (simple) groups $\mathfrak{G}$ such that conjugate type rank of $\mathfrak{G}$ are equal to r. When r increases, this problem probably will become more difficult with exponential growth rate. If, however, the shape of $C(\mathfrak{G})$ is given and coincident with that of the conjugate type graph of some known simple group, then the problem will become considerably tractable.

In previous papers we proved the following theorems:

(I) [7] A finite group $\mathfrak{G}$ is a simple group of the conjugate type

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