J. Math. Kyoto Univ., (JMKYAZ)
13-1 (1973) 159-164

On a ring with a plenty of high order derivations

By

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(Communicated by Professor Nagata, July 31, 1972)

1. Let k and A be commutative rings with 1 and assume that A is a k-algebra. we shall say that the k-algebra A has a plenty of high order derivations over k if the ring of endomorphisms of A over k is filled up with the derivation algebra of A over k, or equivalently any k-linear endomorphism f of A such that $f(1)=0$ is a high order derivation.[1] Such a ring A will be referred to as a P. H. D. ring in the sequel. In the case where both of A and k are fields it was proved in [2] and [4] that A is a P. H. D. ring over k if and only if A is a purely inseparable finite extension of k. The purpose of the present paper is to generalize this result by deleting the assumption that A is a field. The final result is the following

Theorem. *Let k be a field and let A be a commutative k-algebra. Then A is a P. H. D. ring if and only if A satisfies the following three conditions*:

(1) *A is a quasi-local ring.*

(2) *The maximal ideal M of A is nilpotent.*

(3) *The residue field A/M is either k or a purely inseparable finite extension of k.*

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1) Cf [3] for the definition and main properties of high order derivations.