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On a generalization of $\overline{\partial}_b$

By

Masatake KURANISHI

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By meane of the complex of $\bar{\partial}$ exterior derivatives J. J. Kohn and H. Rossi introduced a complex of differential operators $\bar{\partial}_b$ on the smooth boundary of a relatively compact open submanifold in a complex manifold [3]. The subellipticity of $\bar{\partial}$ complex on the part of positive degrees implies the finite dimensionality of the positive degree cohomology of $\bar{\partial}_b$ complex over the closure of the open submanifold. The purpose of the present note is to generalize the above to more general cases of complex of differential operators of degree one, say $(\mathcal{E})$. Firstly we note that the construction of $\bar{\partial}_b$ complex is a special case of a construction valid for any $(\mathcal{E})$ provided the smooth boundary is non-characteristic (with respect to $(\mathcal{E})$). Secondly we show that the subelliptic estimate of $(\mathcal{E})_b$ complex on the part of positive degrees implies the finite dimensionality of the positive degree cohomology of $(\mathcal{E})$ over the closure of the interior of the smooth compact boundary, provided $(\mathcal{E})$ is elliptic. Since it might be useful to generalize the finite dimensionality theorem of cohomology of elliptic complex, we state the exact conditions needed in the proof of the theorem.

It is noted that this generalization of $\bar{\partial}_b$ is also considered by Sweeney in [5].

1. Let Y be a relatively compact open submanifold of a manifold $Y^\sharp$. We assume that Y has the smooth boundary M. Choose once