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A note on Kronecker's "Randwertsatz"

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Kronecker brought to light the determinative properties of certain sets of prime numbers for algebraic number fields and their invariants in his "Ueber die irreductibilität von Gleichungen" which was dedicated to Kummer on his 70th birthday celebration. Kronecker's assertion was called "Randwertsatz" of algebraic number theory by M. Bauer because of Kronecker's statement——

"es ist also (in ähnlicher Weise, wie nach dem Cauchy'schen Satze einen Function durch ihre Randwerte bestimmt wird) mit blossen Congruenzbestimmungen der ganze Inbegriff der durch die Gleichung definierten algebrasschen Irrationalitäten bestimmt".

Since then the base of Kronecker's assertion has been amplified into Frobenius-Tschebotareff's dencity theorem, Bauer's theorem and Gaßmann's theorem, and furthermore his plan has been realized as class field theory in the case of relative abelian number fields. However, in this note, we shall mainly discuss Kronecker-Bauer's "Randwertsatz".

1. Let $\boldsymbol{k}$ be a finite number field, $\Omega/\boldsymbol{k}$ a finite extension. Let $\boldsymbol{K}/\boldsymbol{k}$ be the minimal normal extension containing $\Omega/\boldsymbol{k}$, and let $\boldsymbol{K}'/\boldsymbol{k}$ be the maximal normal extension contained in $\Omega/\boldsymbol{k}$, namely

$$\boldsymbol{K}=\Omega^{(0)}\Omega^{(1)}\cdots\cdots\Omega^{(m-1)},$$

$$\boldsymbol{K}'=\Omega^{(0)}\cap\Omega^{(1)}\cap\cdots\cdots\cap\Omega^{(m-1)},$$

where $\Omega^{(0)}=\Omega$, $m=[\Omega:\boldsymbol{k}]$ and $\{\Omega^{(i)};\ i=0,1,\cdots,m-1\}$ are all the conjugates of Ω over $\boldsymbol{k}$. We define $P(\Omega/\boldsymbol{k})$ and $Q(\Omega/\boldsymbol{k})$ by setting