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Some questions on Cohen-Macaulay rings

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In the present paper, we shall deal with the following type of problem on Cohen-Macaulay rings.

Let $R=\sum_{i=0}^{\infty}R_i$ be a graded noetherian ring, R_i being the module of homogeneous elements of degree i. If R_0 is a field, then it is easy to see that R is Cohen-Macaulay if and only if R_M is Cohen-Macaulay, M being the irrelevant prime ideal (i.e., $M=\sum_{i\geq 1}R_i$).

Our problem is to generalize this characterization.

For instance, one can ask the following questions:

Question 1. Assume that R_M is Cohen-Macaulay for every maximal ideal M such that $M\supseteq\sum_{i\geq 1}R_i$. Does it follow that R is Cohen-Macaulay ?

Question 2. Let K be the total quotient ring of R_0. Then, does the condition that both $R\otimes_{R_0}K$ and R_0 are Cohen-Macaulay imply that R is Cohen-Macaulay ?

To the writer's knowledge, Question 1 is unsolved yet and Question 2 has a negative answer. In the present paper we discuss some facts related to these questions, and main results relate to the case where R is a projective module over R_0.

In connection with these question we ask the following

Question 3. Assume that R is Cohen-Macaulay. Does it follow