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On the propagation of analyticity of solutions of convolution equations

By

Takahiro KAWAI

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The purpose of this note is to prove a theorem on the propagation of the real analyticity of solutions of convolution equations by the simplest way in its nature, i.e., avoiding the explicit use of the *a priori* estimate for convolution operators. (Cf. Kawai [2] §5, where the analogous problem is treated for linear differential operators (of infinite order) with constant coefficients by combining the theory of Fourier hyperfunctions with inequalities for holomorphic functions. See also Kawai [1] Theorem 3.2.1. There a related problem is treated for convolution operators and the method of its proof can be used for our purpose, though we do not employ it here.) The method of the proof given below is essentially the same as that employed in the previous note (Kawai [3]), that is, our proof essentially relies on the theory of microfunctions, especially on the notion of integration along fiber of microfunctions (Sato [5] §6. See also Sato, Kawai and Kashiware [7] Chapter I). Note that the convolution product $\mu*\nu$ of two hyperfunctions μ and ν defined on $\boldsymbol{R}^n$, one of which has compact support, is nothing but the following special kind of integral along fiber of a hyperfunction (Sato [5] § 10)

$$\int\mu(x-y)\nu(y)dy=\int\mu(y)\nu(x-y)dy,$$

which is consistent with that of a microfunction. (Sato [6] § 6)

We first recall the following condition (S) on the convolution