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On uniqueness in some characteristic Cauchy problem for first order systems

By

Akira NAKAOKA

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1. Introduction.

In this paper we shall treat the following system;

$$A(t, x)\frac{\partial \vec{u}}{\partial t} = \sum_{j=1}^{n} B_j(t, x) \frac{\partial \vec{u}}{\partial x_j} + C(t, x)\vec{u}, \tag{1.1}$$

where $A(t, x)$, $B_j(t, x)$ $(j=1,\dots, n)$ and $C(t, x)$ are $N \times N$ matrices whose entries are all analytic in a neighborhood of $(t, x)=(0, 0)$, and $\vec{u}=\vec{u}(t, x)$ is the unknown of N-vector valued function. We consider the Cauchy problem for (1.1) with initial data on the hyperplane $t=0$, and are concerned only with the solution which is analytic in a neighborhood of $(t, x)=(0, 0)$, therefore we use the term "solution" only for analytic solution in what follows.

We assume that the initial plane $t=0$ is characteristic for (1.1), say, $A(t, x)$ is singular at $t=0$. Roughly speaking, the situation will be divided into two cases; one is where $\det A(t, x)$ vanishes only at $t=0$, and another where $\det A(t, x)$ vanishes identically in a neighborhood of the origin.

As for the former case, Y. Hasegawa [4], defining the notion of double characteristic, was mainly concerned with the existence of the analytic solution for single equations. And M. Miyake [5] showed that her method was applicable to some first order systems.

Our interest here is concentrated to the latter case, and we are concerned only with the uniqueness of solution. In our case we can