J. Math. Kyoto Univ. (JMKYAZ)
14-2 (1974) 243-255

On the fields generated by certain points of finite order on Shimura's elliptic curves*

By

Masatoshi YAMAUCHI

(Received, July 12, 1973)

As the title indicates our object of study is an abelian variety B (in the present paper, we are interested only in the one-dimensional case), which was investigated by Shimura [5], [6]. Using such an abelian variety B, he has shown some important relation between the arithemtic of real quadratic fields and the cusp forms of "Neben"-type in Hecke's sense. Here we repeat the result briefly. B is defined over a real quadratic field $k=\boldsymbol{Q}(\sqrt{q})$, whose transform B^{ε} by the non-trivial automorphism ε of k is isogenous to B. Such a B can be obtained from the eigen-function $f(z)=\sum_{n=1}^{\infty} a_n e^{2\pi i n z}$ for all Hecke operators acting on the space $S_2(\Gamma_0(q), \chi)$ of cusp forms of "Neben"-type of weight 2. The eigen-values of Hecke operators for $S_2(\Gamma_0(q), \chi)$ are closely ocnnected with the reciprocity law in certain abelian extenions of k, moreover, such extensions can be generated by the coordinates of some specific section point (c-section point in [6, Th. 2.2. p. 141]) of B. It was observed that two rational integers c and $\mathrm{tr}_{k/\boldsymbol{Q}}(\varepsilon_q)$ have non-trivial common factors where ε_q is the fundamental unit of k [6, § 3] and the Fourier coefficients a_p of $f(z)$ has a certain congruence property with respect to c. As a continuation of this theory, the same investigation was made for the space of cusp forms of "Haupt"-type, by Doi and the present author [2].

* This work was partially supported by the Sakkokai Foundation.