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On functional dimensions of group representations II Case of compact semi-simple Lie groups

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§0. Introduction.

Let E be a σ-Hilbert space topologized by a sequence of norms $\{\|\cdot\|_n, n=1,\ldots\}$. A. N. Kolmogorov has defined the functional dimension $d_f(E)$ of E as follows:

$$d_f(E)=\sup_n \inf_m \lim_{\varepsilon\to 0} \frac{\log H(\varepsilon U_n, U_m)}{\log\log 1/\varepsilon} - 1 ,$$

where $H(\varepsilon U_n, U_m)$ is the ε-entropy of U_m with respect to the norm $\|\cdot\|_n$. If, in particular, E is a space of functions on a compact manifold M, of a certain type, it is known that the functional dimension $d_f(E)$ of E is in close connection with the dimension of M. (Y. Kômura [9] and S. Tanaka [16])

Give a Lie group G acting on M as a group of differentiable transformation, then we can define the representation $\mathfrak{T}=(T_g; g\in G, L^2(M))$ by means of these transformations. Throughout this paper the general σ-Hilbert space is taken to be the space $\mathscr{B}(\mathfrak{T})$ of analytic functionals of the representation $\mathfrak{T}$ in the sense of E. Nelson [12], and calculate the functional dimension of this space. Let $\mathfrak{T}$ be one of the following representation:

1) The regular representation of a connected compact semi-simple Lie group G (in this case the manifold M is the group G itself);