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Sheaf cohomology theory on harmonic spaces

By

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The theory of harmonic functions has been extensively developed since M. Brelot introduced the axiomatic method. Brelot's axiomatic theory consists of a (complete) presheaf $\mathscr{H}$ of vector spaces of continuous functions such that there exist sufficiently many open sets for which Dirichlet problem is solvable, and such that Harnack's principle is satisfied for $\mathscr{H}_U$ on any open set U. But B. Walsh is the first who adapted the general sheaf theory to the study of harmonic functions. B. Walsh, as in the case of classical potential theory, investigated the cohomology groups of $\mathscr{H}$ (or $\mathscr{H}$ with certain limitation at infinity). He also proved, in the presence of the adjoint sheaf $\mathscr{H}^*$ of $\mathscr{H}$, a fundamental duality relation between $\mathscr{H}_c^1(X, \mathscr{H})$ and $\mathscr{H}_X^*$.

In this paper we shall study the theory of duality and cohomology of the sheaves $\mathcal{O}$ on a Brelot's harmonic space that are obtained from $\mathscr{H}$ by limiting it at infinity. (This is the general scheme of solution sheaves of an elliptic second order differential equation with various boundary conditions.) $\mathcal{O}$ is a sheaf on the one-point compactification $X \cup \{a\}$ of a Brelot's harmonic space $(X, \mathscr{H})$ such that, (i) $\mathcal{O}|X = \mathscr{H}$ (ii) there is a neighborhood system of a formed by open sets ω such that any continuous function on the boundary of ω is uniquely extended to a section of $\mathcal{O}$ on ω. $\mathcal{O}$ is no more a sheaf of continuous functions, and a germ in $\mathcal{O}_a$ is recognized as a local solution near the boundary of the above elliptic differential equation.

In section 1 we shall construct various resolutions of $\mathcal{O}$. One