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On Tanaka's imbeddings of Siegel domains

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Introduction

Let D be a Siegel domain of the second kind in $\mathbf{C}^N$. In the case where D is homogeneous, Tanaka [7] showed that there exists an imbedding h of $\mathbf{C}^N$ onto an open subset of a certain complex homogeneous space G_c/B such that every holomorphic transformation of D can be extended to a holomorphic transformation of G_c/B. One of the purposes of this paper is to obtain the same results as Tanaka's without the assumption of homogeneity of D, which is discussed in §2 and §3.

By using the imbedding h, we shall prove in §4 that every holomorphic transformation of D which leaves the Silov boundary of D invariant is an affine automorphism of D. This fact is stated in Pyatetski-Shapiro [5] in the case where D is of the first kind.

Finally, in §5, we shall see that D is a symmetric homogeneous domain if and only if the space G_c/B is compact.

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§1. The automorphisms of Siegel domains

1.1. Let R (resp. W) be a real (resp. complex) vector space of finite dimension. Denote by R_c the complexification of R. For every vector $z \in R_c$, we denote by $\mathrm{Re}\, z$ the real part of z and by $\mathrm{Im}\, z$ the imagi-