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On Finsler spaces with Randers' metric and special forms of important tensors

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In 1941 G. Randers introduced first a special Finsler metric

$$ds=(g_{ij}(x)dx^i dx^j)^{1/2}+b_i(x)dx^i,$$

in a viewpoint of general relativity [12]*. Since then many physicists have developed the general relativity based on this metric. (See References of [5]).

From the standpoint of Finsler geometry itself Randers' metric is very interesting, because its form is simple and properties of the Finsler space equipped with this metric must be described by the ones of the Riemannian space equipped with the metric $L(x, dx)=(g_{ij}(x)dx^i dx^j)^{1/2}$ together with the 1-form $\beta(x, dx)=b_i(x)dx^i$. For example the curvature tensors R_{hijk}, P_{hijk} and S_{hijk} of the Finsler space must be written in terms of Riemannian tensors, that is, the curvature tensor, b_i and its covariant derivatives with respect to the Riemannian connection. But we have few papers concerned with the Finsler space in viewpoint of Finsler geometry [4], [5], [10], [13]. This situation seems to come from the fact that we must hit at once against insuperable difficulty of exhausting calculations to obtain the concrete form of Cartan's $\Gamma^{*i}_{j\ k}$.

The purpose of the present paper is to write the torsion and curvature tensors of the *Randers space* (the Finsler space equipped

* Numbers in brackets refer to the references at the end of the paper.