J. Math. Kyoto Univ. (JMKYAZ)
14-3 (1974) 461-476

Degenerate parabolic differential equations: Necessity of the well-posedness of the Cauchy problem

By

Masatake MIYAKE

(September 26, 1973)

§1. Introduction

We study in this note the following forward Cauchy problem;

$$\frac{\partial}{\partial t}u(x, t)=\sum_{j=0}^{2m}t^{n_j}\mathscr{L}_{2m-j}\left(x, t;\frac{\partial}{\partial x}\right)u(x, t), \tag{1.1}$$

$$u|_{t=0}=u_0(x)\in\mathscr{D}^{\infty}_{L^2}(R^n_x), \tag{1.2}$$

where $\mathscr{L}_{2m-j}\left(x, t; \frac{\partial}{\partial x}\right)=\sum_{|\alpha|=2m-j}a_{\alpha,j}(x, t)\left(\frac{\partial}{\partial x}\right)^{\alpha}$, $a_{\alpha,j}(x, t)\in\mathscr{E}^0_t(\mathscr{B}_x)$[1], $(x, t)\in R^n_x\times[0, 1]$ and $n_j\geqq 0$.

Our purpose in this note is to seek a necessary condition of the $\mathscr{D}^{\infty}_{L^2}$-well-posedness for the Cauchy problem (1.1)-(1.2). Recently K. Igari [4] has studied this problem, but our research is different from it. For instance, our research is based on the *modified order*[2] of the

1) $\mathscr{D}^{\infty}_{L^2}(R^n_x)=\left\{u(x);\left(\frac{\partial}{\partial x}\right)^{\alpha}u(x)\in L^2(R^n_x)\text{ for any }\alpha\right\}$

$\mathscr{B}_x(R^n_x)=\left\{u(x)\in C^{\infty}(R^n_x);\left|\left(\frac{\partial}{\partial x}\right)^{\alpha}u(x)\right|\leqq M_{\alpha}\text{ for some }M_{\alpha}\geqq 0\text{ for any }\alpha\right\}$

$u(x, t)\in\mathscr{E}^0_t(\mathscr{B}_x)$ means that $u(x, t)\in\mathscr{B}_x$ for any fixed t and continuous in t in the usual topology of $\mathscr{B}_x$.

2) We say that the modified order at $t=0$ of $t^{n_j}\mathscr{L}_{2m-j}$ is $\frac{2m-j}{n_j+1}$.