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A necessary condition for well-posed Cauchy problems

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§ 0. Introduction

Consider the partial differential equation

$$L[u]=\left(\frac{\partial}{\partial t}\right)^m u+\sum_{|\nu|+j\leq m,\, j<m} a_{\nu j}(x, t)\left(\frac{\partial}{\partial x}\right)^\nu\left(\frac{\partial}{\partial t}\right)^j u=0 \tag{0.1}$$

in $\Omega=\boldsymbol{R}^n\times[0, T]$, $T>0$. Consider the Cauchy problem for this equation with given initial data at $t=0$.

Definition. *We say that the Cauchy problem for* (0.1) *is well-posed in* $\mathscr{D}_{L^2}^\infty$, *if for any given initial data* $\varphi_j(x)\in\mathscr{D}_{L^2}^\infty$, $j=0, 1,\dots, m-1$, *there exists a unique solution* $u(x, t)\in\mathscr{E}_t^m(\mathscr{D}_{L^2}^\infty)$, $0\leq t\leq T$, *which takes the given initial data at* $t=0$:

$$\left(\frac{\partial}{\partial t}\right)^j u(x, t)|_{t=0}=\varphi_j(x), \qquad j=0, 1,\dots, m-1\,.$$

It is an interesting problem to look for a necessary condition in order that the Cauchy problem be well-posed, and this problem was studied by many authors. The following theorem proved by S. Mizohata is one of the most important results.

We consider the characteristic equation

$$p(\lambda)=\lambda^m+\sum_{|\nu|+j=m} a_{\nu j}(x, t)\xi^\nu\lambda^j=0, \qquad \xi\in\boldsymbol{R}^n\,. \tag{0.2}$$