

Infinitesimal automorphisms on the tangent bundle

By

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1. Introduction

Let M be an n -dimensional differentiable manifold of class C^∞ and TM its tangent bundle, which is a $2n$ -dimensional differentiable manifold. The prolongation of tensor fields and connections from M to TM has been studied in [3] by Yano and Kobayashi. In particular, for an affine connection ∇ on M , its complete lift ∇^C is an affine connection on TM . If g is a pseudo-Riemannian metric on M , its complete lift g^C is a pseudo-Riemannian metric on TM with n positive and n negative signs.

In [4], Yano and Kobayashi have tried to determine the form of an infinitesimal affine transformation on (TM, ∇^C) . However, their work is incomplete because they have determined essentially only the fibre-preserving infinitesimal affine transformations. In the same paper, they have also tried to determine the form of an infinitesimal isometry on (TM, g^C) . But their result turned out to be incorrect as was pointed out by Tanno [1], who in turn gave a complete solution on the form of an infinitesimal isometry on (TM, g^C) .

In this paper, we shall use the method of adapted frames to determine the most general form of an infinitesimal affine transformation on (TM, ∇^C) , without any extra assumption on the infinitesimal affine transformation itself. For the case of fibre-preserving transformations, our result is an improvement over that given in [4]. As an application of our results and further illustration of our method, we shall give an alternative proof of the result of Tanno on infinitesimal isometries on (TM, g^C) mentioned earlier.

2. Preliminaries

In this section, we shall summarize all the basic definitions and results that are needed later. Most of them are well-known, and details can be found in Yano [2] and Yano and Kobayashi [3, 4]. Indices $a, b, c, \dots; h, i, j, \dots$ have