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Integral cohomology ring of the symmetricspace *EII*

By

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§1. Introduction

The simply connected Riemannian symmetric spaces have been classified. For classical cases, their cohomology rings are well known. For exceptional cases, those of ***FII, EIII, EIV*** and ***EVII*** are known [1], [14], [15], and in these cases they are torsion free. The remaining spaces ***G, FI, EI, EII, EV, EVI, EVIII*** and ***EIX*** have 2-torsions, and the cohomology rings of the first two are known [5], [12].

The purpose of this paper is to determine the integral cohomology ring of the compact Riemannian symmetric space ***EII***. As a homogeneous space, ***EII*** is expressed by $E_6/S^3\cdot SU(6)$, where E_6 is the compact 1-connected exceptional Lie group of rank 6 and $S^3 \cap SU(6)=\boldsymbol{Z}_2$ [12].

In order to determine $H^*(\boldsymbol{EII})$, we first consider a homogeneous space E_6/C, where $C=T^1\cdot SU(6)$ is the centralizer of a one-dimensional torus.

Our first result is

Theorem 3.2. $H^*(E_6/C)=\boldsymbol{Z}[t, u, v, w]/(r_{12}, r_{16}, r_{18}, r_{24})$,

where $\deg t=2$, $\deg u=6$, $\deg v=8$, $\deg w=12$, *and*

(2.6) $$r_{12}=u^2+2w-3vt^2-ut^3+2t^6,\quad r_{16}=t^8+3wt^2-3v^2.$$

$$r_{18}=2wu-wt^3 \quad \textit{and} \quad r_{24}=w^2+26v^3-15v^2t^4-21wvt^2+9wut^3.$$

Using the Gysin exact sequence for the S^1-bundle: $E_6/SU(6)\to E_6/C$ we have

Corollary 3.5. $H^i(E_6/SU(6))\cong\boldsymbol{Z}$ *for* $i=0, 6, 8, 12, 14, 20, 23, 29, 31, 35, 37, 43$; $\cong\boldsymbol{Z}_2$ *for* $i=18, 26$; $\cong\boldsymbol{Z}_3$ *for* $i=16, 28$ *and* $=0$ *for the other* i.