J. Math. Kyoto Univ. (JMKYAZ)
17-2 (1977) 259-298

Hopf algebra structure of simple Lie groups

By

Akira KONO

(Received May 25, 1976)

§1. Introduction

Let $\boldsymbol{G}$ be a compact connected Lie group of rank l and p a rational prime. The group multiplication $\mu: \boldsymbol{G}\times\boldsymbol{G}\to\boldsymbol{G}$ induces a map

$$\mu^*: H^*(\boldsymbol{G};\ \boldsymbol{Z}_p) \longrightarrow H^*(\boldsymbol{G}\times\boldsymbol{G};\ \boldsymbol{Z}_p). \tag{1.1}$$

By the virtue of the Künneth formular, μ^* gives a Hopf algebra structure

$$\phi: H^*(\boldsymbol{G};\ \boldsymbol{Z}_p) \longrightarrow H^*(\boldsymbol{G};\ \boldsymbol{Z}_p)\otimes H^*(\boldsymbol{G};\ \boldsymbol{Z}_p) \tag{1.2}$$

of $H^*(\boldsymbol{G};\ \boldsymbol{Z}_p)$.

Since

$$\phi(x)-(x\otimes 1+1\otimes x)\in \tilde{H}^*(\boldsymbol{G};\ \boldsymbol{Z}_p)\otimes\tilde{H}^*(\boldsymbol{G};\ \boldsymbol{Z}_p)$$

$$\text{for}\quad \tilde{H}^*(\boldsymbol{G};\ \boldsymbol{Z}_p)=\sum_{i>0} H^i(\boldsymbol{G};\ \boldsymbol{Z}_p),$$

we put

$$\bar{\phi}(x)=\phi(x)-(x\otimes 1+1\otimes x).$$

An element $x\in\tilde{H}^*(\boldsymbol{G};\ \boldsymbol{Z}_p)$ is said to be primitive if $\bar{\phi}(x)=0$.

On the other hand consider the universal $\boldsymbol{G}$ bundle

$$\boldsymbol{G}\longrightarrow \boldsymbol{EG}\longrightarrow \boldsymbol{BG}. \tag{1.3}$$

An element $x\in\tilde{H}^*(\boldsymbol{G};\ \boldsymbol{Z}_p)$ is called to be universally transgressive if x is transgressive with respect to (1.3).