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A sharp form of the existence theorem for hyperbolic mixed problems of second order

By

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Sadao MIYATAKE

§1. Introduction

In this paper we consider the following initial-boundary value problem (P) which we denote also by $\{P, B\}$

$$(P) \qquad \begin{cases} Pu=f(x, t), & \text{for } x\in\Omega,\ t\in R^1_+, \\ Bu|_S=g(s, t), & \text{for } s\in S,\ t\in R^1_+, \\ D_t^j u|_{t=0}=u_j(x),\ (j=0, 1), & \text{for } x\in\Omega, \end{cases}$$

in the cylinder domain $\Omega\times(0, \infty)$, where Ω is the exterior or interior of a smooth and compact hypersurface S in R^{n+1}. P is a regularly hyperbolic operator of second order and S is non characteristic to P. Moreover we assume that the only one of $\tau_1(\nu)$ and $\tau_2(\nu)$ is negative for all $(s, t)\in S\times(0, \infty)$, where $\tau_1(\xi)$ and $\tau_2(\xi)$ are the roots of $P(s, t, \xi, \tau)=0$ and ν is the inner unit normal at s. This assumption means that the number of boundary conditions is one[1]. Therefore we assume $P(s, t, 0, 1)<0$ and $P(s, x, \nu, 0)=1$. B is a first order operator:

$$B(s, t\ D_x, D_t)=\sum_{j=1}^{n+1} b_j D_{x_j}-cD_t, \qquad \left(D_t=\frac{1}{i}\frac{\partial}{\partial t}, \text{ etc.}\right),$$

and we suppose $B(s, t, \nu, 0)=1$.

We assume that all the coefficients of P and B are smooth and that they

1) This assumption is equivalent to the condition that only one root $q(\eta, \tau)$ of $P(s, t, q\nu+\eta, \tau)=0$ has positive imaginary part for $\mathrm{Im}\ \tau<0$. (cf. p. 121 in [3].) In fact, to see this, we may consider the case $\eta\equiv0$, taking account of the hyperbolicity of P.