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Formal fibers and openness of loci

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Introduction

Many loci are Zariski open for a large class of rings (algebro-geometric, analytic, complete, excellent) and such openness of loci is variously related to the good properties of formal fibers. To quote the well known examples, the geometric regularity of formal fibers implies, for a noetherian local ring A, the openness of regular locus for Spec (A'), where A' is any A-algebra of finite type, while the geometric reduceness of fibers carries the openness of normal locus.

The converse arrow is also true for some class of rings: for instance, if A is complete for some $\mathfrak{m}$-adic topology and excellent modulo $\mathfrak{m}$, then the openness of regular locus implies the geometric regularity of formal fibers (see [12], theorem 4).

In the present paper we investigate fibers and loci for a property $\boldsymbol{P}$ meaningful in any noetherian ring, submitted to the following conditions:

1—every field has $\boldsymbol{P}$;

2—$\boldsymbol{P}$ is local;

3—if A is a complete local ring, then the $\boldsymbol{P}$-locus of A is Zariski open;

4—if $(A, \mathfrak{m})\rightarrow(B, \mathfrak{n})$ is a faithfully flat local homomorphism, then $\boldsymbol{P}$ descends from B to A; if moreover $B/\mathfrak{m}B$ has $\boldsymbol{P}$, then $\boldsymbol{P}$ ascends;

5—if A is regular, then A has $\boldsymbol{P}$.

In n. 1. after a short recall on the main properties we need in the paper (Cohen-Macaulay, Gorenstein, complete intersection), we discuss the openness of $\boldsymbol{P}$-locus on a ring A and on finite A-algebras, giving a list of examples.

In n. 2 we discuss the so called "Nagata's criterion for the openness of loci", formally the same as the criterion for the openness of regular locus, but concerning a property $\boldsymbol{P}$ of the type considered above.