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Plessner points, Julia points, and ρ^*-points[1)]

By

J. S. Hwang

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1. Introduction.

Let $f(z)$ be a function defined in the unit disk $D(|z|<1)$. As in [14], a point $e^{i\theta}$ on the unit circle $C(|z|=1)$ is called a Plessner point of f provided each angular cluster set $C(f, e^{i\theta})$ of f at $e^{i\theta}$ coincides with the extended plane. Following [6], we call a point $e^{i\theta}$ a Julia point of f provided in each Stolz angle Δ having one vertex at $e^{i\theta}$ the function f assumes all values on the Riemann sphere except possibly two. For $z, z' \in D$, we denote by $\rho(z, z')$ the non-Euclidean metric $\rho(z, z') = \frac{1}{2}\log[(1+a)/(1-a)]$, where $a = |(z'-z)/(1-\bar{z}z')|$. We call $\rho(z, z')$ the ρ-distance between z and z'. As in [9], a sequence $\Delta(n)$ of disks in D is called a sequence of cercles de remplissage for f provided that the ρ-diameters of $\Delta(n)$ tend to zero, and the images $f(\Delta(n))$ cover all of the Riemann sphere, with the possible exception of two sets $E(n)$ and $G(n)$ whose spherical diameters tend to zero as $n\to\infty$. The sequence $\{z_n\}$ of centres of the disks $\{\Delta(n)\}$ is called a sequence of ρ-points for f. A point $e^{i\theta}$ is called a ρ^*-point of f provided each Stolz angle Δ with one vertex at $e^{i\theta}$ possesses a sequence of ρ-points of f.

The content of this article has six more sections. In section 2, we discuss the inclusion property among Plessner points, Julia points, and ρ^*-points. Then we present a sufficient condition of normal functions in section 3. In section 4, we construct some holomorphic

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