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On the structure of minimal surfaces of general type with $2p_g=(K^2)+2$

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Introduction. Let S be a minimal nonsingular projective surface of general type defined over an algebraically closed field k of characteristic 0. We denote by p_g and K_S, respectively the geometric genus and the canonical divisor of S. In a series of papers [5], [6] and [7], Horikawa studied the structure (the number of moduli, the deformation type, etc.) of minimal nonsingular projective surfaces S of general type satisfying the equality: $2p_g=(K^2)+3$ or $2p_g=(K^2)+4$. The surfaces studied by Horikawa are, however, the extreme cases in the sense that if the value of p_g is given, $(K^2)=2p_g-3$ or $2p_g-4$ is the smallest possible value of (K^2) (cf. [3], Theorem 9). In the present article, by employing the methods introduced in [5] and used effectively in [6] and [7], we shall study the structures of minimal nonsingular projective surfaces of general type satisfying the equality $2p_g=(K^2)+2$, of which we shall give a description under several mild restrictions. In the first section of the present article, various results are collected, which we use below frequently and sometimes without specified references. In the second section we prove that the irregularity q vanishes for minimal surfaces of general type with $2p_g=(K^2)+2$. In the third and fourth sections we have to limit ourselves to the case where $|K|$ has no fixed component. This assumption implies that $|K|$ is not composed of a pencil. On the other hand, $|K|$ has at most two base points. In the third section we consider the case where $|K|$ has no base point and $n:=p_g-1\geqslant 3$. Then, morphism $\varphi:=\Phi_{|K|}: S\longrightarrow V\subset \boldsymbol{P}^n$ (where $V:=\varphi(S)$) defined by $|K|$ is a morphism of degree 2 except when $n=3$ and $\deg\varphi=3$. If one assumes that $n\geqslant 3$ and $\deg\varphi=2$ then V is a Del Pezzo surface of degree n; thus $n\leqslant 9$. The