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Formal degree and Clebsch-Gordan coefficient

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1. Let G be a unimodular locally compact group, and $\omega=\{\mathfrak{H}, U_g\}$ be a unitary representation of G. Here $\mathfrak{H}$ is the space of representation ω and U_g's are its representation operators.

We call ω L^2-representation if and only if ω is irreducible and there exists a non-zero vector v in $\mathfrak{H}$ such that $<U_g v, v>$ is a square integrable function of g in G with respect to the right Haar measure dg on G.

For an L^2-representation ω, the following properties are known (cf. [1]).

1) For any vectors u, w in $\mathfrak{H}$, $<U_g u, w>$ is square integrable.

2) For a fixed non-zero vector v in $\mathfrak{H}$, the map

$$\mathfrak{H} \ni u \longrightarrow <U_g u, v> \in L^2(G)$$

is an intertwining operator from ω to the right regular representation $\mathfrak{R}=\{L^2(G), R_g\}$ of G.

3) For any representation $\tau=\{\mathfrak{K}, V_g\}$ which is disjoint to ω, and any vectors u, v in $\mathfrak{H}$, any vectors x, y in $\mathfrak{K}$ for which $<V_g x, y>$ is square integrable,

$$\int_G <U_g u, v>\overline{<V_g x, y>}dg=0.$$

4) There exists a positive number $d(\omega)$, depending only on ω, such that

$$\int_G <U_g u, v>\overline{<U_g w, z>}dg=d(\omega)^{-1}<u, w><z, v>$$

for any u, v, w, z in $\mathfrak{H}$.