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Cardinals, isols, and the growth of functions

By

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1. Introduction

Let ω be the non-negative integers and let Λ_Z be the cosimple isols. To each $X\in\Lambda_Z$ we can associate a unique degree of unsolvability Δ_X (c. f. [3]). This is the degree of any co-r. e. $\xi\in X$. Throughout this paper $\boldsymbol{d}$ is a non-recursive r. e. degree and Λ_d is the set $\{X\in\Lambda_Z \mid \Delta_X\leq\boldsymbol{d}\}$. In this paper we are concerned with the first order theory of $(\Lambda_d, +)$ where $+$ is isolic addition. We study this structure by means of a first order language L containing individual variables $u_0, u_1, \ldots, v_0, v_1, \ldots,$ a binary functor $+$ denoting addition, and a binary predicate $=$ denoting equality. L is interpreted in ω or Λ_d in the usual way. Because ω and Λ_d are commutative semigroups we take the liberty of putting of L in the normal form $\sum_{i<n} a_i u_i$ where $\sum$ denotes summation, $a_i\in\omega$, and $a_i u_i$ is the term consisting of u_i summed with itself a_i times. An *AE special Horn sentence* is a sentence of L having the form

$$(1)\qquad (\forall u_0, \ldots, u_{m-1})(\alpha\rightarrow(\exists v_0, \ldots, v_{n-1})\beta)$$

where $\alpha(u_0, \ldots, u_{m-1})$ has the form

$$(2)\qquad \bigwedge_{j<q}\left(\sum_{k<m} a_{jk}u_k=\sum_{k<m} a'_{jk}u_k\right)$$

and $\beta(u_0, \ldots, u_{m-1}, v_0, \ldots, v_{n-1})$ has the form

$$(3)\qquad \bigwedge_{j<r}\left(\sum_{k<m} b_{jk}u_k+\sum_{k<n} c_{jk}v_k = \sum_{k<m} b'_{jk}u_k+\sum_{k<n} c'_{jk}v_k\right).$$

In [6] it is shown that