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Uniqueness of the factorization under composition of certain entire functions

By

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Introduction

After the classical works of G. Julia [19] and P. Fatou [9], [10] on the iteration and composition theory for polynomials or rational functions, I. N. Baker has investigated the theory in the case of transcendental entire functions since 1955 and obtained many results. In particular, he generalized the minimum modulus theorem concerning entire functions of order less than 1/2 ([2] Theorem 3) and further proved, using Fatou's theory of iteration, interesting theorems concerning the permutability of transcendental entire functions ([2], [3], [4]). In 1968, F. Gross [13] and M. Ozawa [24] proved independently that certain entire functions do not have any factorization (by composition) into trascendental entire factors. Since then, there have appeared many results in factorization theory, by applying Nevanlinna theory etc. However, most of these recent results (except [21], [26]) concern the impossibility of factorization, that is, the primeness, the pseudoprimeness and so on.

In this paper, we shall treat certain composite functions of two or three prime functions, which belong to certain special classes. For the functions of these classes one can show the forms of their factors (Theorems 1 and 2, proved first by S. Koont [21], except one of the conclusions in Theorem 1). We shall give a simpler proof of these two theorems in § 2. Using these facts as key lemmas, we shall proceed to prove our main Theorems 3, 4, 5, 6 and 7, which assert that the factorization by composition of certain entire functions is