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# On the generalized local cohomology and its duality

By

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## § 0. Introduction.

Let $(R, \mathfrak{M}, K)$ be a commutative, Noetherian local ring with the non-zero multiplicative identity and all the modules considered be unitary throughout.

The main purpose of this paper is to study the generalized local cohomology $H^i_{\mathfrak{M}}(*, *)$ introduced by J. Herzog:

$$H^i_{\mathfrak{M}}(M, N) = \varinjlim_{m} \mathrm{Ext}^i_R(M/\mathfrak{M}^m M, N)$$

for R-modules $M$ and $N$, [5] (1. 1. 1). This is in fact a generalized one of the usual local cohomology $H^i_{\mathfrak{M}}(*)$ : for any R-module $N$, $H^i_{\mathfrak{m}}(R, N) = H^i_{\mathfrak{M}}(N)$.

As is well known, the vanishing (or non-vanishing) of the local cohomology module $H^i_{\mathfrak{M}}(N)$ of a finitely generated (abbreviated to f.-g. from now on) R-module $N$ reflects some important character of $N$, say dimension and depth of $N$. It is quite reasonable to ask when the generalized local cohomology module $H^i_{\mathfrak{M}}(M, N)$ of R-modules $M$ and $N$ vanishes (or never.)

Our first result, Theorem (2. 3), states that the lower bound of i's for which $H^i_{\mathfrak{M}}(M, N) \neq 0$ (for f.-g. non-zero modules $M$ and $N$ over $R$) coincides with the $\mathrm{depth}_R N$. As to the upper bound, we must require some restrictions on either $M$ or $N$: for all sufficiently large i's $H^i_{\mathfrak{M}}(M, N) = 0$ if and only if either $Pd_R(M) < \infty$ or $Id_R(N) < \infty$, (2. 4). Where $Pd_R$ (resp. $Id_R$) denotes the *projective* (resp. *injective*) *dimension* over $R$. We shall mainly treat the case when $Pd_R(M) < \infty$ in this paper.