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Rigidity for isometric imbeddings

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Introduction.

Let $\boldsymbol{f}$ be an immersion of a manifold M into the m-dimensional Euclidean space $\boldsymbol{R}^m$. Assume that $\boldsymbol{f}$ is non-degenerate (see § 1). In his paper [15] one of the authors has shown that there is associated to $\boldsymbol{f}$ a linear differential operator L in a natural manner, and that it is equivalent, in a sense, to the differential operator $d\Phi_f$ of infinitesimal isometric deformations of $\boldsymbol{f}$ ([15], Theorem 1.2; Theorem 1.1 in the present paper). Especially the infinitesimal isometric deformations $\boldsymbol{u}$ of $\boldsymbol{f}$ are in a one-to-one correspondence with the solutions φ of the equation $L\varphi=0$. It should be here pointed out that the symbol of the operator $d\Phi_f$ necessarily degenerates, while the symbol of the operator L does not necessarily degenerate. Thus we have the notion of ellipticity for the operator L. These facts indicate that the equation $L\varphi=0$ plays an important role in the study of the rigidity problem for the immersion $\boldsymbol{f}$.

Owing the operator L, he has indeed established a global rigidity theorem ([15], Theorem 2.4) which may be stated as follows: Let $\boldsymbol{f}_0$ be an immersion $M\rightarrow\boldsymbol{R}^m$ which satisfies the following conditions: 1) $\boldsymbol{f}_0$ is elliptic, i. e., $\boldsymbol{f}_0$ is non-degenerate and the associated equation $L\varphi=0$ is elliptic; 2) $\boldsymbol{f}_0$ is globally infinitesimally rigid, i. e., every global solution of the equation $L\varphi=0$ is derived from an infinitesimal Euclidean transformation of $\boldsymbol{R}^m$; 3) M is compact. Then the theorem states that if two imbeddings $\boldsymbol{f}$ and $\boldsymbol{f}'$: $M\rightarrow\boldsymbol{R}^m$ lie both near to $\boldsymbol{f}_0$ with respect to the C^3-topology, and if they induce the same Riemannian metric g, then there is a unique Euclidean transformation a of $\boldsymbol{R}^m$ such that $\boldsymbol{f}'=a\boldsymbol{f}$. He has also applied this theorem to the canonical isometric imbedding $\boldsymbol{f}_0$ of a compact hermitian symmetric space, $M=K/K_0$, into the Euclidean space $\mathfrak{k}=\boldsymbol{R}^m$, $\mathfrak{k}$ being the Lie algebra of K, and has obtained a rigidity