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On Hartshorne's conjecture

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§0. Introduction

After studying ample vector bundles on algebraic varieties, R. Hartshorne has posed the following problem in [5] and now it is known as the conjecture of Harshorne's.

(H–n) If X is an n-dimensional non-singular projective algebraic variety with ample tangent vector bundle defined over an algebraically closed field k, then X is (algebraically) isomorphic to $\mathbf{P}^n$ over k.

In the case $k=\mathbf{C}$ (the complex number field), it is known that this conjecture is deeply connected with the following famous conjecture of Frankel's in complex differential geometry.

(F–n) A compact Kaehler manifold X of dimension n with positive sectional curvature is biholomorphic to the complex projective space $\mathbf{P}^n(\mathbf{C})$.

From now on, we assume that the characteristic of k is 0. (H–1) and (F–1) are obvious. Using classification of algebraic surfaces, (H–2) and (F–2) are solved affirmatively by R. Hartshorne [5] and by Frankel and Andreotti [3] respectively. Recently, T. Mabuchi has succeeded in proving (H–3) under the assumption that the second Betti number of X is equal to 1 [9]. In this paper, we will prove that (H–3) holds true without the assumption on the second Betti number. The keys to our proof of (H–3) are the following.

(1) A criterion for $\mathrm{Pic}(X)=\mathbf{Z}$: Let X be a non-singular projective algebraic variety with ample anti-canonical divisor $c_1=c_1(T_X)$. Then the Picard number $\rho(X)$ of X is equal to 1 if and only if every effective divisor on X is ample (Theorem 3). Using this criterion, we prove that if the tangent vector bundle T_X of X is ample, then the Picard number $\rho(X)$ of X is equal to 1 (Theorem 4).

(2) A characterization of projective spaces: If a non-singular projective algebraic variety X has a non-zero global vector field vanishing on an ample irreducible effective divisor D on X, then X is isomorphic to a projective space $\mathbf{P}^n$ and D cor-

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