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On the Cauchy problem for some non-kowalewskian equations with distinct characteristic roots

By

Jiro TAKEUCHI

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1. Introduction.

Consider a linear partial differential operator

$$P(x\,;D_x,\,D_t)=D_t^m+a_1(x\,;D_x)D_t^{m-1}+\cdots+a_m(x\,;D_x),\ (x,\,t)\in\boldsymbol{R}^l\times[0,\,T]\equiv\Omega \tag{1.1}$$

where $a_i(x\,;D_x)$ $(1\leqq i\leqq m)$ is a linear partial differential operator in $\boldsymbol{R}^l$.

It is said that $P(x\,;D_x,\,D_t)$ defined by (1.1) is non-kowalewskian if

$$\max_{1\leq j\leq m}\ \text{order}\ a_j(x\,;D_x)/j\equiv b>1\,. \tag{1.2}$$

Denote the homogeneous part of order jb of $a_j(x\,;D_x)$ by $a_j^0(x\,;D_x)$.

$$P^0(x\,;\xi,\,\tau)=\tau^m+a_1^0(x\,;\xi)\tau^{m-1}+\cdots+a_m^0(x\,;\xi) \tag{1.3}$$

is said to be the principal symbol of $P(x\,;D_x,\,D_t)$. $D_t=-i\dfrac{\partial}{\partial t}$, $D_x=-i\dfrac{\partial}{\partial x}$.

Consider the forward and backward Cauchy problem

$$\begin{cases} P(x\,;D_x,\,D_t)u(x,\,t)=f(x,\,t) & \text{on}\quad \Omega \\ D_t^ju(x,\,t_0)=g_j(x)\,, \quad j=0,\,1,\,\cdots,\,m-1 & \text{for any}\quad t_0\in[0,\,T]\,. \end{cases} \tag{1.4}$$

As is well known, it is necessary for the forward and backward Cauchy problem (1.4) to be H^∞-wellposed that the characteristic equation in τ $P^0(x\,;\xi,\,\tau)=0$ has the only real roots for any $(x,\,\xi)\in\boldsymbol{R}^l\times\boldsymbol{R}^l$. (cf. Petrowskii [4] and Mizohata [3]). As a corollary it follows from H^∞-wellposedness that $b=\max\{\text{order}\ a_j/j\,;1\leqq j\leqq m\}$ is an integer if we assume that $b>1$.

Denote the characteristic roots by $\lambda_j(x,\,\xi)$, i.e.

$$P^0(x\,;\xi,\,\tau)=\prod_{j=1}^m(\tau-\lambda_j(x,\,\xi))\,. \tag{1.5}$$

From now on we only consider the case where $b=2$.

We shall give sufficient conditions for the forward and backward Cauchy problem to have a unique solution in $L^2(\boldsymbol{R}^l)$.

We assume the following conditions.