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The total energy decay of solutions for the wave equation with a dissipative term

By

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§ 1. Introduction and the result.

Let Ω be an open domain $\subset \boldsymbol{R}^n (n \geqq 1)$ exterior to a smooth bounded closed surface $\partial\Omega$. We shall consider the exterior initial-boundary value problem of the following type:

$$L[u]=u_{tt}(x, t)+a(x, t)u_t(x, t)-\Delta u(x, t)=0, \tag{1.1}$$

where $t \geqq 0$, $x=(x_1, x_2, \cdots, x_n) \in \Omega$, $u_{tt}=\frac{\partial^2 u}{\partial t^2}$, $u_t=\frac{\partial u}{\partial t}$, $\Delta u=\sum_{k=1}^{n} \frac{\partial^2 u}{\partial x_k^2}$ and $a(x, t)$ is non-negative;

$$u(x, 0)=f(x) \quad \text{and} \quad u_t(x, 0)=g(x), \tag{1.2}$$

where $f(x)$ and $g(x)$ are real-valued continuous functions with compact support contained in the ball of radius ρ centered at the origin and $f(x)$ belongs to class C^1;

$$u(x, t)=0 \text{ on } \partial\Omega \quad \text{or} \quad \frac{\partial u}{\partial n}(x, t)=0 \text{ on } \partial\Omega, \tag{1.3}$$

where $\frac{\partial}{\partial n}$ denotes the outward normal derivative on $\partial\Omega$.

The assumptions on the dissipative term $a(x, t)$ of (1.1) will be stated precisely afterwards.

Let $u=u(x, t)$ be a real-valued smooth solution of (1.1), (1.2) and (1.3). We define the total enegy $E(t)$ and $E(0)$ for u as follows.

$$E(t)=\int_{\Omega}\{|u_t(x, t)|^2+|\nabla u(x, t)|^2\}\, dx$$

and

$$E(0)=\int_{\Omega}\{|u_t(x, 0)|^2+|\nabla u(x, 0)|^2\}\, dx$$

$$=\int_{\Omega}\{|g(x)|^2+|\nabla f(x)|^2\}\, dx=\|g\|^2+\|\nabla f\|^2,$$